Quantum Entanglement Coexists with Causality

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Introduction

There are many articles about quantum entanglement and causality on the Internet, if not in peer-review physics journals. But it seems as if very few or none of the articles on the Internet explain simply why quantum mechanical entanglement does not violate the principle of causality. This is the idea that the cause of an effect must precede the effect.

The following is for those with an undergraduate degree in physics or mathematics or an equivalent background. A short bibliography is given at the end, citing only publications, not URLs.

No claim to originality is made. The ideas here can almost certainly be found elsewhere, such as the idea at the end of Section V that the uniqueness of the arrow of time is lost in entanglement. I am unaware who first showed that quantum entanglement does not violate causality. I have not read elsewhere the idea at the end of Section V that entanglement information apparently has zero rest-mass.

I. The Problem

In 1979, *Scientific American* published an article by d'Espagnat on quantum mechanical entanglement.[1] It said that in 1935 Einstein, Podolsky, and Rosen (EPR) questioned quantum mechanics because it predicted quantum entanglement, a phenomenon that could not be real.[2] The word "entanglement" was not in use in 1935 and is nowhere in the article. The EPR paper is the only case where Einstein published his deep doubts about the meaning of quantum mechanics.

It went on to say that in 1964 J. S. Bell published an inequality governing the statistics of quantum mechanical experiments that could test whether quantum entanglement were real. To get the inequality, Bell assumed that objective reality exists, logic works, and nothing travels faster than c, the speed of light in vacuum. The key aspect of Bell's inequality is that it disagrees with the quantum theory of entangled particles.

In 1979, the jury was still out, but initial experimental results were in. Quantum entanglement is delicate and very hard to test. But the majority of experimental results indicated that quantum entanglement was real and hence one of Bell's three assumptions might be wrong. Some long-held belief might have to go.

The reality of quantum entanglement is now confirmed. Which of the three assumptions is wrong? It is the third. d'Espagnat guessed that it might be.

In 2009, Albert and Galchen wrote an article in *Scientific American* that seemed to say there is still a problem.[3] There is an apparent paradox, but no actual problem.

The apparent paradox is this: Special Relativity says that nothing with a positive rest-mass can reach c and that no message, or signal, or something carrying information, can travel faster than c. Specifically, if a message carrying information could travel from point A to point B at a speed greater than c in one rest-frame, then causality would be violated in some other rest-frame, as discussed in Section II.

In quantum entanglement, however, a message does travel faster than c, as described in Section III. Not only faster than c, but a message can travel from one entangled particle to the other instantaneously, no matter how far apart they are and no matter what lies between them. The speed of the message is infinite. The particles need no physical connection or link. They communicate telephathically. They aced Basic Magic 101.

Quantum entanglement is applied magic. It strains credulity. It could be the strangest trick of nature, even stranger than pair-production in vacuum. But it is a natural part of quantum mechanics.

At first sight, quantum engtanglement should violate causality. But it cannot. Causality must be inviolable. We cannot accept the notion that the effect of some random event, such as the toss of a coin, whose effect is "Heads" or "Tails," can come before the event itself. How does quantum entanglement not violate causality? How can quantum entanglement coexist with causality?

The answer is trivial. It is known to every serious physicist. It is obvious. It only took me a year to see it. I have yet to read a simple explanation of it, but it must be explained hundreds of times in every tongue. Of course, whether an idea is obvious depends on one's frame of reference.

The professor had covered three blackboards with part of a long proof thirty minutes into the class. Joe raised a hand. "Professor, is it obvious that the first equation implies the second?" The professor looked at the first two equations, paused, and then walked out. Just before the end of the hour, he returned. "Oh, yes, it is perfectly obvious."

Section II shows why a massive, or physical, message traveling faster than c would violate causality, a standard textbook problem.[4] I use a thought experiment in which a coin is tossed in the caboose of a train. The outcome and the time when it occurred in the eyes of the train (that is, the time shown on the caboose clock) are written on a slip of paper. The slip of paper is thrown towards the engine at a speed greater than c in the eyes of the ground. This leads to a violation of causality in the eyes of the train. A reader needs familiarity with the Lorentz transform, Eqs. (1) and (2) below, to follow the proof.

Section III describes quantum entanglement. I use an analogy between a pair of quantum mechanically entangled particles and a pair of magically linked coins. Section IV shows why quantum entanglement does not violate causality.

The fundamental idea is that either entangled particle can be observed first or (equivalently) interact with another particle first. In the analogy, either coin can be tossed first. This distinguishes quantum entanglement from the toss of single coin. It is an escape hatch. For a single coin, the toss is the unique cause and must precede the effect, "Heads" or "Tails." But with entanglement, the cause is not unique. This evades what would otherwise be a problem when the order of cause and effect is reversed.

Another basic idea is that entanglement transmits information, rather than a massive, or physical body. This evades the speed limit of c on mechanical grounds.

II. A Faster-than-Light Message Violates Causality

Let one reference frame be the ground and the other be a train. In the eyes of the ground, the train moves to the right along the x-axis at speed v, where $0 < v < c$. The spatial and temporal coordinates of an event in the ground frame are denoted, respectively, by x and t and in the train frame by x' and t'. The caboose is at the origin $x' = 0$ of the train frame.

Let $\beta \equiv v/c$ and $\gamma \equiv 1/\sqrt{1-\beta^2}$. If the origins of the frames coincide when both clocks at the origin of each frame of reference read the time 0, then the Lorentz transform equations are

$$x' = \gamma(x - \beta ct)\,, \tag{1}$$

$$ct' = \gamma(ct - \beta x)\,. \tag{2}$$

From Eq. (2), if the distance between the caboose and the engine is L in the ground's eyes, then the engine clock lags the caboose clock by the amount $\gamma\beta L/c$ regardless of t.

If one sets $x = ct/\beta = (c/\beta)t$ in Eq. (2), then $t' = 0$. Hence some train clock reads $t' = 0$ at any ground time t. It lies farther and farther to the right along the x-axis as time goes by. The point on the x-axis where it lies moves with speed $c/\beta \equiv v_0$. It is a different train clock for every value of t. Although v_0 exceeds c, it need not exceed c by much. As $\beta \to 1$, $v_0 \to c$, meaning that v_0 might exceed c only by an infinitesimal amount.

Let the caboose toss a coin when its own clock reads $t' > 0$. Have someone write the time t' on a slip of paper along with the outcome "H" or "T." If the toss is "Heads," then the slip of paper might read, "At caboose time t', the coin landed showing H."

Suppose that this slip of paper is given speed v_m (for "message") along the positive x-axis in the eyes of the ground. If $v_m > v_0$, then this slip of paper will eventually overtake some train clock that reads $t' = 0$. It is moving faster than the coordinate of the train clock that reads $t' = 0$. But v_m need only exceed c infinitesimally.

To visualize the situation concretely, and with no loss of generality, we can let the clock that reads $t' = 0$ when the slip of paper arrives be the engine clock at the front of the train. We have deduced that the engine would get a message from the future. It predicts the result of a random event that has not occurred yet in its eyes. The engine clock shows the time $t' = 0$, but the time on the slip of paper from the caboose is $t' > 0$.

This violates causality. One cannot know with certainty the outcome of a random event yet to occur. We cannot get a slip of paper from the future. A thing like a slip of paper with a message on it cannot break the speed limit of c.

Of course, nothing like a slip of paper with a positive rest-mass can reach the speed c for another reason. Its mass in the eyes of the ground would become infinite as its speed approached c. But even if a slip of paper were massless, the argument above shows that causality forbids it to travel faster than c.

III. Quantum Entanglement

When an excited atom decays, without a change of its angular momentum, a pair of entangled photons can be emitted in opposite directions. By conservation of angular momentum, the sum of the angular momenta of these photons is equal to zero. The reader may be familiar with quantum mechanical spin. If one particle is spin-up, the other must be spin-down. (If the thumb of your right hand points up, your fingers naturally curl in the counterclockwise direction, corresponding to positive angular momentum or spin along your thumb.) Electrons are better to use than photons in experiments, but various types objects have been entangled.

According to quantum mechanics, the first particle to be measured or to interact with another particle does not make up its mind which spin to have until it is actually measured or interacts with another particle. It is random. That is, the spin of each is indefinite before this. Mathematically, the quantum mechanical wave function of the two-particle system is the (normalized) sum of the two wave functions of the two possible states that may result from measurement or interaction.

If “$+$” represents an up-state and “$-$” represents a down-state for two entangled particles called 1 and 2, then the possible two-particle states for particles 1 and 2 can be written as $<+,->$ and $<-,+>$. The first and the second symbol are for particle 1 and 2, respectively. The wave function of the two-particle system is proportional to the sum of these two states. But if one particle decides, by our experiment or by interacting with another particle, to be spin-up, then the other is forced to be spin-down. Only one of the states survives.

This is exactly like tossing two coins minted simultaneously in a single processes that magically links them. They fly apart, but by magic they have the property that if one has landed showing “Heads,” then the other will land showing “Tails.” It does not matter how far apart the coins are. Either can be tossed first. The outcome of the first toss is random. The second toss can be made as soon as the first toss is made.

This is nearly unbelievable. It implies that a signal must go from the first coin to the second instantaneously, telling it what to do, no matter how far apart they are and no matter what lies betweeen them. No physical connection is needed. A similar problem, if not the same, occurs with the so-called “collapse of the wave function.”

The quantum mechanical wave function of an electron, left to its own devices, extends throughout all space, in general. That is, it does not vanish anywhere. It specifies the likelihood, no matter how small, that the electron will be found at one location or another at one time or another when forced to reveal its location.

If an electron is detected in a small volume of space, then its wave function collapses, meaning that it becomes highly localized. How can the part of the wave function near Uranus or in the Horsehead Nebula know that it has to vanish in time for the wave function to collapse and resemble a Dirac delta function in three dimensions? Quantum mechanics texts do not address this. [5–8]

Has the information that a measurement or interaction is about to occur flown over an infinite distance in no time at all? Is everything foreordained? What is going on? Probably no one has an answer.

IV. Causality is Preserved

Let one entangled coin be in the caboose and the other in the engine. Suppose the caboose tosses its coin first in the eyes of the ground. If it lands showing “Heads,” then the engine coin must land showing “Tails.”

Recall that the engine clock lags the caboose clock in the ground's eyes. One might think, then, that the engineer gets a faster-than-light message from the caboose (and from the future) before having tossed the engine coin. The message predicts the outcome of a random coin-toss, and so violates causality. It seems as if the engineer got a slip of paper, in effect.

This reasoning is wrong. First, the coin receives the message, not the engineer. The engineer does not receive something akin to a slip of paper with a message from the future about the outcome of a random event that has not yet occurred. The engineer has no idea that the outcome of the toss of the engine coin is dictated.

Moreover, if there were in fact a slip of paper thrown from the caboose at a speed greater than c, the engineer could have tossed the engine coin before the slip of paper arrived. The engineer need not have waited for it.

In the eyes of the ground, the engine coin can be tossed as soon as the caboose coin is tossed. Since the engine clock lags the caboose clock in the eyes of the ground, this toss would occur before the toss in the caboose in the eyes of the train. In this situation, in the eyes of the train, the coin-toss in the engine is the cause and that in the caboose is the effect. The cause precedes the effect.

It is not necessary that the engine coin be tossed. But it could have been tossed. The engine coin made its decision before the engine clock reads $t' = 0$, regardless of whether the engineer knows this.

How can causality be violated? It seems impossible to violate causality in the situation described here, since neither toss is the unique cause. To the ground, the coin toss in the caboose is the cause, and that in the engine is the effect. To the train, the toss in the engine is the cause, and that in the caboose is the effect. In each case, cause precedes effect.

V. Remarks

Feynman discusses the EPR paper in *The Feynman Lectures on Physics.*[9] He is not distressed by it. He also discusses charge conservation as an example of local conservation laws. Reference 10 contains Bell's articles and reference 11 is a popular book (that is, for the general public) on quantum entanglement.

Simultaneity is lost in special relativity. That is, in Eq. (2), t' is different for a single value of t for different values of x. But if two events, a cause and its effect, are causally related in the ground frame, then their time-ordering cannot be reversed in the train frame. This, too, is a standard problem (or is a comment) in texts. For clarity, by definition, "causally related" means that, if the spatial separation of the events is

$x \geq 0$ and the time that elapses between them is $t > 0$, then $x \leq ct$. One may verify this (it is a good exercise) using Eqs. (1) and (2).

Marburger describes quantum entanglement as "deeply disturbing."[12] He does not say, "deeply distressing," but only "deeply disturbing." He does not dwell on it at great length.

Section IV noted that, in the special case of quantum entanglement, no one can say which of two events is the cause of the other in an absolute sense. It depends entirely on which rest-frame observes the events. This means that nature has lost the uniqueness of the arrow of time.

Quantum entanglement also would seem to imply that entanglement information has zero rest-mass. Otherwise, its mass would become infinite when it reached c. But in fact, entanglement information travels at an infinite speed.

BIBLIOGRAPHY

1. B. D'espagnat, "The Quantum Theory and Reality," *Sci. Am.*, November, 1979, 158-181. This is excellent. It is detailed, but you need not follow everything in it to understand this note.

2. A. Einstein, B. Podolsky and N. Rosen, "Can a Quantum Mechanical Description of Reality be Considered Complete?", *Phys. Rev.*, **47**, 777-780,

1935. If you do not understand the argument, you are in good company.
3. D.Z. Albert and R. Galchen, “A Quantum Threat,” *Sci. Am.*, March, 2009, p. 32-39. This article claims that the Theory of Special Relativity is threatened by quantum entanglement. It is unclear what the problem is, but perhaps it is nonlocality.
4. R. Resnick, *Introduction to Special Relativity*, John Wiley & Sons, New York (1968), problem 39, page 106.
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10. J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press, New York (1993). According to reference 11, this contains Bell’s papers.

11. Amir D. Aczul, *Entanglement: The Greatest Mystery in Physics*, Penguin Group, Plume, New York (2003).

12. J.H. Marburger, *Constructing Reality*, Cambridge University Press, Cambridge (2011), pages 144-152. This is an excellent popular book. It has a derivation, shorter than that in Reference 1, of a version of Bell's inequality different from that in Reference 1. It notes that the Standard Model is not a model, in that we have no mental picture of how nature works quantum mechanically, and that the word "particle" to describe electrons and the like is a misnomer. The footnotes alone make the book worthwhile.