Quantum Entanglement Coexists with Causality

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A well known paradox involves quantum mechanical entanglement and causality. An internet search on "quantum mechanical entanglement and causality" gets more than 200,000 hits. Causality merely means that a cause must precede its effect. In all these articles and discussions, it seems impossible to find a simple proof that quantum entanglement coexists with causality. One might never expect that there is a simple proof. This note has a self-contained, simple proof.

This note has background for readers who have not studied Special Relativity or quantum mechanics. The proof itself is not given until Section IV. Algebra appears, but a general reader should be able to follow the logic. One need only imagine people measuring times and distances with clocks and rulers, but it would be necessary to read carefully.

The ideas here are not apt to be original. For example, Section V notes that the uniqueness of the arrow of time is lost in entanglement and that entanglement information has zero rest-mass.

I. The Problem

Bernard d'Espagnat had a fascinating article in *Scientific American* in 1979.[1] It said that in 1935 Einstein, Podolsky, and Rosen (EPR) questioned quantum mechanics because it predicted quantum entanglement, a phenomenon that defied belief.[2] This was the only paper that Einstein published on his deep doubts about quantum theory. The word "entanglement" was not in use in 1935, by the way.

d'Espagnet said that in 1964 J. S. Bell published an inequality governing the statistics of quantum mechanical experiments. The striking fact was that Bell's inequality contradicted the quantum theory of entangled particles. Testing this inequality in experiments could reveal whether quantum entanglement were real.

Bell's inequality rested on three assumptions: 1) Objective reality exists. Things are not just in your mind. Reality is out there. 2) Logic works. If statement A implies statement B, then if statement A is true, then so is B, and if B is false, so is A. 3) Nothing, especially information, may travel faster than c, the speed of light in vacuum.

Quantum entanglement is exquisitely delicate and hard to test. In 1979, the jury was still out, but initial results were in. Experiments indicated that quantum entanglement was real. One of Bell's assumptions might be wrong.

Some long-held, fundamental tenet of physics might have to be abandoned.

Quantum entanglement is now established. Which assumption was wrong? The third, as d'Espagnat guessed. In 2009, *Scientific American* published an article by Albert and Galchen saying that there is still a problem.[3] There is only an apparent paradox.

The paradox: A standard textbook problem[4] in Special Relativity is that no message, signal, or anything carrying information, can travel faster than c. Why? Because, if a hypothetical message carrying information could go from point A to point B at a speed greater than c in one frame of reference, or rest-frame, such as the ground, then causality would be violated in another frame of reference, such as a train. A proof is given in Section II.

Section III discusses entanglement. In quantum entanglement, a message does travel faster than c. In fact, the message has an infinite speed. It goes from one entangled particle to the other instantaneously. This happens no matter how far apart they are. Either particle can send the message. It does not matter what lies between them. They have no physical connection. They must communicate telepathically. They aced Magic 101.

Quantum entanglement is magic. It strains credulity. It could be the strangest trick of nature, even stranger than vacuum pair-production.

Because quantum entanglement involves a message that travels faster than light, it would seem to violate causality. We can't allow that. Causality must hold. The effect of a random event, like the toss of a coin, whose effect is Heads or Tails, certainly cannot be known before the event itself. How does quantum entanglement manage not to violate causality?

The solution is trivial. It must be known to all physicists. It only took me a year to see it. Still, the treatments of which I am aware[5,6,7] are more complicated than my solution. Of course, whether a thing is obvious may depend on the frame of reference.

Halfway through the lecture, the professor had filled three blackboards with part of a long proof. Joe asked, "Professor, is it obvious how the first equation implies the second?" The professor paused, stared at the equations, and then walked out of the lecture hall. He returned at the last second. "Oh, yes, it's perfectly obvious."

I will mention briefly how the proof goes in Section II that a message or signal traveling faster than c would violate causality. The proof uses a thought-experiment in which a coin is tossed in the caboose of a train. The outcome, "H" or "T," and the time when the coin was tossed in the eyes of the train, meaning the time shown on the caboose-clock, are written on a slip of paper.

It is assumed that this slip can be thrown towards the engine at a speed greater than c in the eyes of the ground. It turns out that this leads to a violation of causality in the eyes of the train: the outcome of the toss is known before the toss occurs in the train's eyes. Hence it is impossible to throw the slip at a speed great than c. Assuming that we could leads to an absurdity. This is "proof by contradiction."

Section III first describes quantum entanglement. It then introduces an analogy between two entangled particles and a pair of magically linked coins. Section IV shows why quantum entanglement does not violate causality.

The main idea is simply that either entangled particle can be observed first. Equivalently, either may interact with another particle first. In the analogy, either coin can be tossed first. This is the escape hatch. It distinguishes quantum entanglement from the toss of a single coin.

For a single coin, the toss is the unique cause. It must precede the effect of Heads or Tails. In contrast, with entanglement the cause is not unique. This eliminates the problem with causality that arises for a single coin, namely that the time-order of two events is different in the eyes of the ground and in the eyes of the train.

Entanglement transmits information, rather than a massive, or “physical” body. This lets us break the speed limit of c on a massive body in Special Relativity. We return to this point at the end of Section II.

II. A Faster-than-Light Message Violates Causality

The idea of frames of reference is introduced shortly along with notation. First, for the general reader, it should be said that the Theory of Special Relativity flows from a single assumption, the Principle of Relativity: all reference frames are created equal. That is, they see the same laws of physics. This assumption is egalitarian. It says that nature does not play favorites.

The Principle of Relativity means that there is no way to detect absolute motion. The motion of one frame of reference is only motion relative to some other frame of reference. You may recall being on a train when the trees and buildings were hidden from view. If the train next to you is moving with respect to you, you cannot tell whether you or that other train is moving with respect to the ground.

The Principle of Relativity also means that a blob of light observed by one frame of reference, such as a train, has the same speed c in its eyes as it has in the eyes of any other frame of reference, such as the ground.

This theory says that time and distance are not absolute quantities. The time elapsed between two events and the distance between them depend on which frame observes the events. In this theory, a time is the number on the face of a clock and a distance is the number of tick-marks between two positions on a meter-stick. By definition, time and distance are just numbers that come from basic observation and measurement.

Let the ground be one reference frame and a train be the other. In the eyes of the ground, let the train move to the right along the x-axis at constant speed v, with $0 < v < c$ (v lies between 0 and c). The position and time of an event in the ground frame are denoted, respectively, by x and t and in the train frame by x' and t'. The caboose is at the origin, meaning $x' = 0$, in the train frame.

Each frame has fixed observers at rest relative to it. These observers are lined up left and right along the tracks on the ground and from caboose to engine on the train. The observers in each frame have identical clocks at rest next to them. In the eyes of each frame, its own clocks are synchronized. A method of synchronizing clocks is discussed in Section V.

A set of meter-sticks is at rest in each frame, laid end-to-end along the

x-axis. For any event, each frame's observers can record the times shown on the clocks and the distances shown on the meter-sticks adjacent to them in the other frame, as well as the times and distances on their own. They can also compare notes or share data after an experiment.

Meter-sticks let the observers measure distances. For time, think of stop-watches instead of normal clocks. We only care about how much time elapses between events and a stop-watch can show a time equal to 0, which is useful. It can even show a negative time, which is just some time before the hand or the digital display of the stop-watch reaches 0.

Following tradition, let $\beta \equiv v/c$ and $\gamma \equiv 1/\sqrt{1-\beta^2}$. ($\beta$, "beta," is the second letter of the Greek alphabet and γ, "gamma," the third.) We can assume for simplicity that the origins of the frames coincide when the clock at each origin shows time 0. Then the Lorentz transform equations are

$$x' = \gamma(x - \beta ct)\,, \tag{1}$$

$$ct' = \gamma(ct - \beta x)\,. \tag{2}$$

These follow logically from the Principle of Relativity. This might not be obvious, nor is it obvious what they even mean. You need not worry about those things, but it is worth knowing that Eqs. (1) and (2) are a pinnacle of intellectual achievement and a deep truth of nature in a realm far beyond experience.

The Lorentz transform was known before Einstein published on Special Relativity in 1905, but it took him ten years to develop the theory, starting at age sixteen.

The Principle of Relativity is simple and profound. By adopting it, we evade an unanswerable question: "How does light know how fast to go in empty space?" We simply *assume* that any given blob of light has the same measured speed in the eyes of any rest-frame, or frame of reference, regardless of their motion relative to each other.

This is completely at odds with intuition. It means that distance and time are not what you thought they were before you met Special Relativity.

Equations (1) and (2) take us from the ground's eyes (no primes) to the train's eyes (primes). For any chosen values of x and t, the space-time coordinates (x, t) are an "event." They can be pictured as Cartesian coordinates on perpendicular axes (the "abscissa" and "ordinate") in a two-dimensional space-time plane. An event is a point in this plane.

The equations say, "The space-time point or event (x, t) in the ground frame is at the space-time point or event (x', t') in the train frame." The primed points, or events, overlie the unprimed ones in space-time. They are in one-to-one correspondence.

The results we need come from Eq. (2). It says that if L is the distance between the caboose and the engine in the ground's eyes, then the engine-clock lags the caboose-clock by the amount $\gamma\beta L/c$ regardless of t. You can see this. Substitute $x = L$ into Eq. (2) to get an expression for the train time in the engine. Then substitute $x = 0$ into it to get an expression for the train-time in the caboose. Finally subtract the first time from the second. The time t will cancel out.

The second result is that if $x = ct/\beta = (c/\beta)t$ in Eq. (2), then $t' = 0$. This means that some train clock reads $t' = 0$ at any ground time t. It lies farther and farther to the right along the x-axis as time goes by. The x-coordinate of this clock has speed $v_0 \equiv c/\beta > c$. It is a different train clock for every value of t. Although v_0 exceeds c, it need not exceed c by much. If $\beta \to 1$, then $v_0 \to c$. Therefore v_0 may exceed c only infinitesimally.

Suppose that the caboose tosses a coin when its own clock reads $t' > 0$ (greater than 0) and writes on a slip of paper both the time t' and the outcome "H" or "T." If the coin lands showing Heads, then the slip of paper might read, "At caboose time $t' = 3$ minutes, the coin landed showing H."

Suppose that this slip of paper is given speed v_m (for "message") along the positive x-axis in the eyes of the ground. If $v_m > v_0$, then this slip of paper will eventually overtake some train clock that reads $t' = 0$. It is moving faster than the coordinate of the train clock that reads $t' = 0$. Note that v_m need only exceed c infinitesimally, because v_0 need only exceed c only infinitesimally.

To visualize this more concretely, consider the engineer. Suppose that the train clock that reads $t' = 0$ when the slip of paper arrives is in the engine car at the front of the train. We have just deduced that when the engine-clock shows the time $t' = 0$, the engineer gets a slip of paper from the caboose about an event that occurred at a time $t' > 0$ in the caboose's eyes. To the train, and in particular the engine, this is a message from the future. It predicts the outcome of a random event, but that event has not yet occurred as far as the engineer is concerned.

This means that causality has been violated. Causality says that one cannot know the outcome of a random event yet to occur. One cannot get a slip of paper from the future. Therefore the premise of our argument must have been false. That is, a slip of paper carrying a message cannot exceed the speed limit c. When we assumed that it could, we were led by perfectly logical steps, imagining people making observations, to an absurdity. As mentioned, this is “proof by contradiction” (in Latin, *reductio ad absurdum*).

If you have studied Special Relativity, then you know that nothing like a slip of paper with a positive rest-mass can reach the speed c. A body’s mass approaches infinity as its speed approaches c. That theorem is an aside for us. Even if a slip of paper were massless, the argument above says it cannot have a speed greater than c.

III. Quantum Entanglement

Atoms have discrete energy levels, or “states,” like the rungs of a ladder, but not equally spaced. An excited atom decays to a state of lower energy when one of its excited electrons falls to a vacant state of smaller energy. This is accompanied by the emission of energy in the form of light. The emitted light is a particle called a “photon.” An atom can also absorb a photon. This kicks an electron up to a state of larger energy.

In some decays, an atom emits a pair of photons that fly away in opposite directions. It can do this without spinning like a top. Angular momentum is conserved. It does not change over time, or rather it does not change in this particular case. Hence the total angular momentum of the two photons must be equal to zero. A pair of electrons can also be emitted from a single process in such a way that the sum of their angular momenta is zero.

If the photons were wheels, they would have to rotate in opposite directions. To make another analogy, consider someone standing at the center of a merry-go-round or sitting on a bar-stool. If the merry-go-round or the bar-stool has frictionless bearings, then if that person threw two spinning discs in opposite directions, and were not spinning after having thrown them, then the discs would necessarily spin in opposite directions.

Angular momentum, or spin, can be positive or negative according to a certain convention. Point the thumb of your right hand vertically upward. Let the thumb be the z axis, and let z increase in the upward direction. The fingers of your right hand will curl counterclockwise. (You are presumably looking down at your hand.) This goes with a counterclockwise sense of rotation, which means positive angular momentum or "spin" along the z-axis. If your thumb points down, the rotation is clockwise. Then the angular momentum or spin along the z-axis is negative. This is the common, traditional definition of the sign of spin. It is called the "right-hand rule."

Here are some basic and strange facts about the angular momentum, or spin, of photons and electrons. All photons have the same angular momentum or spin, as do all electrons. The spin of a photon is 1 and that of an electron is 1/2, expressed in units of Planck's constant divided by 2π.

The spin of elementary particles is most strange. The spin of a photon or an electron is always either up or down along any axis that you choose. It does not matter in what direction your axis points. Space is isotropic and spin is quantized. Particles are incomprehensible.

Quantum mechanics says that the first of the two entangled particles to be measured or to interact with another particle does not make up its mind which spin to have until it is measured or interacts with another particle. It is random, indefinite, and unpredictable, and equally likely to be spin-up or spin-down.

Regarding mathematics, quantum mechanics represents the two-particle system, before either particle is observed or has interacted with another particle, as the (normalized) sum of the two possible vectors, called "state-vectors," that can result from a measurement or interaction. One method for this uses algebra as follows:

Let $|+>$ represent the up-state and $|->$ the down-state for a single particle. The symbols here are column vectors called "Dirac ket-vectors." For two entangled particles, say 1 and 2, the possible two-particle states after an observation or interaction may be written $|+,->$ and $|-,+>$. The first and the second symbol are for particle 1 and 2, respectively.

The initial state of the two-particle system, before observation or interaction, is an equal 50-50 mixture of the two states $|+,->$ and $|-,+>$, that is, their sum or difference, within an overall multiplicative constant. Once either particle decides to be spin-up (or spin-down), either because we carried out a measurement on it or because it interacted with a third particle, the other is forced to be spin-down (or spin-up). The state changes and becomes one of the two-particle states. Only one state survives.

That sketches, very briefly, the birth, behavior, and one mathematical representation of entangled particles. We provide little discussion or explanation, because the behavior of entangled particles is all we care about. We now can make an analogy between entangled particles and everyday objects which are simple to think about.

A pair of entangled particles is analogous to two coins that are minted simultaneously by a method that magically links them. They fly apart, but by magic if one lands Heads, then the other will land Tails. It does not matter how far apart the coins are, either can be tossed first, the outcome of the first toss is random, and the second coin can be tossed immediately after the first.

Is this believable? Does nature allow this? Nothing could be weirder. A message or signal goes from the first coin to the second instantaneously, telling it what to do. This holds no matter how far apart they are, no matter what lies betweeen them, and with no physical connection between them. A similar phenomenon, if not the same one in different garb, is the "collapse of the wave function."

The quantum mechanical wave function of any one electron contains everything that can be said about it. It varies with position and time, and, left on its own, extends throughout all space. It does not vanish (equal zero) anywhere. It specifies the likelihood, or more precisely the probability per cubic centimeter, that the electron will appear at any given location at any given time. Einstein did not believe nature was random at heart.

If an electron is found in a tiny volume, then its wave function collapses, becoming highly localized. Outside of that volume, it is zero. How does the part of the wave function near Uranus or in the Horsehead Nebula know that it has to vanish in time for the wave function to collapse? Texts ignore this obvious question.[8–11]

Has the information that a measurement or interaction is about to occur flown over an infinite distance in no time at all? Is everything foreordained? What is going on? No one knows. Quantum mechanics makes no sense, but "maintains its perilous but still correct existence."[12]

IV. Causality is Preserved

The question is whether entanglement lets any observer see the effect of a cause prior to the cause. Is the proper time-order of cause and effect upset in the eyes of some reference frame? The question is not whether quantum entanglement is a real phenomenon nor whether a message can travel infinitely fast. It can.

Let one entangled coin be in the caboose and the other in the engine. Suppose the caboose tosses its coin when its own clock reads $t' = 0$, and in the eyes of the ground, the engine-coin has not yet been tossed. If the caboose-coin lands Heads, then the engine-coin must land Tails.

As seen by the ground frame, the engine-clock lags the caboose-clock. According to the ground, when the caboose-clock reads $t' = 0$, the engine-clock reads $-\gamma\beta L/c$, where L is the length of the train in the ground's eyes.

It might seem from this that the engineer receives a faster-than-light message from the caboose, and from the future, before the engine-coin is tossed, that predicts the outcome of that toss. This seems to violate causality. It seems as if the engineer got a slip of paper, in effect, as in Section II.

It is not that way at all. The engineer does not get a slip of paper. The coin receives the message. The engineer does not know that the outcome of the coin-toss in the engine is determined.

In the eyes of the ground, the engine-coin may be tossed immediately after the coin-toss in the caboose. Suppose it were. In the eyes of the train, *the engine-toss would precede the caboose-toss*, because the engine-clock lags the caboose-clock in the eyes of the ground. The engine-clock shows the negative time $-\gamma\beta/L/c$.

This means that, in the eyes of the train, the coin-toss in the engine is the cause of the outcome of the coin-toss in the caboose. To the train, this outcome is the effect, and the cause would precede the effect. Causality is not violated. There is no paradox, and in fact the resolution of the apparent paradox is trivial.

Causality cannot be violated since neither toss is the unique cause of the outcome of the other toss. To the ground, the coin-toss in the caboose may be the cause, dictating the result of that in the engine. To the train, in the same experiment, the coin-toss in the engine may be the cause, dictating the result of that in the caboose, the effect. In both frames, cause precedes effect, despite the infinite speed of the signal.

V. Remarks

The proof above relied on the fact that the engine-clock lags the caboose-clock in the eyes of the ground. It is worthwhile to see that this follows directly from the Principle of Relativity. At the same time, we can see how to synchronize clocks in a given reference frame.

Imagine that the train people put a flashbulb midway between the caboose and the engine. It will emit a pulse of light going left towards the caboose and another going right towards the engine.

The people on the train cause the caboose-clock and the engine-clock to start ticking, meaning they read time $t' = 0$, when their respective pulses arrive. To the train, the pulses have the same speed c and they travel equal distances. To the train, they arrive simultaneously at the caboose and engine. The synchronization does not need to involve people, of course. It could be done with devices that detect a light pulse.

The ground frame disagrees. In the ground's eyes, the right-going pulse must travel farther to catch the engine than the left-going pulse has to travel to hit the caboose. Why? Because the train is going to the right.

That is, in the ground's eyes, each pulse has speed c, so the left-going pulse hits the caboose before the right-going pulse catches the engine. The caboose-clock must begin ticking before the engine-clock. This is the same as saying that the caboose-clock leads the engine-clock. Equivalently, the engine-clock lags the caboose-clock. As mentioned, the amount of lag is constant.

Feynman[12] discusses the EPR paper. He is not distressed, but he does not mention causality. He also discusses charge conservation to illustrate a local conservation law. Reference 13 has Bell's articles. Reference 7 is a popular book, meaning for the general public. In another popular book, Marburger says quantum entanglement is "deeply disturbing."[14] He does not say, "deeply distressing" nor dwell on entanglement, nor mention causality.

We saw that simultaneity is lost in Special Relativity. That is, in Eq. (2), for a fixed value of t, t' is a different number at different values of x. In the ground's eyes, the train's clocks are not synchronized. One cannot freely play games with time and space, however. In particular, if two events A and B are "causally related" in the ground frame, then they are also causally related in the train frame.

Two events A and B are said to be "causally related" if they are separated by a distance x and by a time t for which $x \leq ct$. This is the same as $x/t \leq c$, which says that one event can send a message to the other in an elapsed time of t at a speed less than or equal to c. Equivalently, if a cause propagated at speed less than or equal to c, then one event could cause an effect at the other in time less than or equal to t.

For "acausal events," $x > ct$. Such events are too far apart for a message to go from one to the other in time t, even at speed c. One event cannot cause the other, unless a cause propagates at a speed greater than c. This happens with entangled particles.

Using Eqs. (1) and (2) to prove the claim above regarding causal events in different frames is a very good (not too hard) high-school algebra problem. Similar problems are to show that a) a clock ticks more slowly (emits fewer ticks per second) when moving than when stationary and b) a train is shorter in length when moving than when stationary, in both cases by a factor of γ.

Section IV noted that, in the special case of quantum entanglement, one cannot say which of two events is the cause of the other in an absolute sense. Instead, it depends on which rest-frame observes them. Nature has therefore lost the uniqueness of the arrow of time. This may suggest a question about entropy.

Entanglement information seems to have zero rest-mass. If it did not, then its mass would become infinite (or imaginary) when it reached (or exceeded) c, according to the Theory of Special Relativity.

You might object to the proof, saying, "Entangled particles cannot ride in the caboose and engine. They always move with opposite velocities, away from their birthplace, not at the same velocity as a train."

The particles or coins were riding in the caboose and engine to make the experiment easy to picture. Their speeds do not matter. They could have been moving with opposite velocities in the ground's eyes, for example, not at rest relative to the train. I assumed only that the particles or coins revealed their states when they happened to be adjacent to the caboose and engine.

As mentioned, the first particle to be observed or to interact with another particle decides randomly and on its own which state to have and which message to send. The experimentalist can force the particle to decide, but is not a party to its choice. This means that one cannot send a "1" or "0" via the magic link.

The link is delicate. You cannot even breath on it. You should avoid looking at it or even thinking too much about it. If you touch it, it is destroyed.

This is true of all coherent quantum mechanical effects. Feynman[12] opens Volume III by discussing an experiment: firing monoenergetic electrons at an opaque plate with two slits in it. On a screen behind the plate, the count of electrons per unit area will vary with position like the interference or diffraction pattern that one would see for waves, as in optical diffraction. Far from the slits, the waves emanating from each slit are in step in some directions, so they add up, and out of step in others, so they cancel each other. Electrons act as if they were sound waves or light waves. The wavelength of any given electron is inversely proportional to its momentum, according to de Broglie's formula, by the way.

This happens even if the electrons are fired at the plate one at a time. It takes longer to get the same statistics, but this shows clearly that the interference pattern is a not collective effect. It is not because electrons interact with each other. Each individual electron acts like a wave.

The pattern disappears if the experiment is designed to determine which slit each electron went through. That determination requires an interaction with each electron, which upsets its wavefunction. It is no longer a plane wave.

An example of the difference between coherent waves with very flat wavefronts and other waves is the twinkle of stars and (nearly) steady glow of planets. Stars are so distant that their wavefronts are flat to within less than one wavelength of light when they hit the atmosphere. Planets are near enough that their light has slightly curved wavefronts. The random variations in the atmosphere that scatter light therefore have a negligible effect on the brightness of a planet, but they cause strong interference patterns when they scatter starlight.

Another example is the shifting, bright and dark speckle pattern of the spot of light from a laser on a sheet of paper. The interference pattern results from the nearly perfect periodicity of the light waves.

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