Quantum Entanglement Coexists with Causality: A Simple Proof Using the Lorentz Transform

Richard C. Shockley

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Many URLs address an apparent paradox in physics involving quantum mechanical entanglement and causality. Causality is the principle that a cause precedes any effect it produces. Few if any of these sites show why quantum entanglement does not violate causality. This note gives a simple proof that quantum entanglement does not violate causality.

I include background material. The proof itself does not appear until Section IV. Algebra is used, but the reasoning should be clear to a careful nonmathematical reader. One need only picture simple, concrete experiments described in plain English.

The proof and other ideas here are not likely original. In Section V, it is noted that the uniqueness of the arrow of time is lost in entanglement and that entanglement information has zero rest-mass.

I. The Problem

Scientific American had a fascinating article by Bernard d'Espagnat in 1979.[1] It said that in 1935 Einstein, Podolsky, and Rosen (EPR) questioned quantum mechanics because it predicted quantum entanglement, a phenomenon that defied belief.[2] This was the only paper that Einstein published on his deep doubts about quantum theory. The word "entanglement" was not in use in 1935, by the way.

d'Espagnet said that in 1964 J. S. Bell published an inequality governing the statistics of quantum mechanical experiments. The striking fact was that Bell's inequality contradicted the quantum theory of entangled particles. Testing this inequality in experiments could reveal whether quantum entanglement were real.

Bell's inequality rested on three assumptions: 1) Objective reality exists. Things are not just in your mind. Reality is out there. 2) Logic works. If statement A implies statement B, then if statement A is true, then so is B, and if B is false, so is A. 3) Nothing, especially information, may travel faster than c, the speed of light in vacuum.

Quantum entanglement is exquisitely delicate and hard to test. In 1979, the jury was still out, but initial results were in. Experiments indicated that quantum entanglement was real. One of Bell's assumptions might be wrong.

Physicists might have to abandon some long-held, fundamental tenet.

Quantum entanglement is now established as real. Which assumption was wrong? The third, as d'Espagnat guessed. In 2009, *Scientific American* published an article by Albert and Galchen saying that there is still a problem.[3] There is only an apparent paradox.

The paradox: A theorem and standard textbook problem[4] in Special Relativity is that no message, signal, or anything carrying information, can travel faster than c. Why? Because, if a hypothetical message carrying information could go from point A to point B at a speed greater than c in one frame of reference, or rest-frame, such as the ground, then causality would be violated in another frame of reference, such as a train. A proof is given, for the sake of completeness, in Section II.

In quantum entanglement, however, a message does travel faster than c, as described in Section III. In fact, the speed of the message in infinite, because it goes from one entangled particle to the other instantaneously. Moreover, it does not matter how far apart they are, nor what lies between them, nor which one sends the message. They have no physical connection. They communicate telepathically. They aced Magic 101.

Quantum entanglement is real magic. It strains credulity. It could be the strangest trick of nature, even stranger than vacuum pair-production.

Quantum entanglement should violate causality. That is not allowed. Causality must hold. We do not believe that the effect of a random event, such as the toss of a coin, whose effect is heads ("H") or tails ("T"), could ever be known before the event occurs. How or why does quantum entanglement manage not to violate causality?

The solution is trivial, utterly obvious, and simple. Once I began to try, it only took a year. It is surely known to any serious physicist and published in every tongue. Of course, whether a thing is obvious may depend on the frame of reference.

Thirty minutes into the lecture, the professor had filled three blackboards with part of a proof. Joe asked, "Is it obvious that the first equation implies the second, Professor?" The professor paused, stared at the first two equations, and walked out of the lecture hall. He returned at the last second. "Perfectly obvious."

To show why a message or signal traveling faster than c would violate causality, I use a thought-experiment in which a coin is tossed in the caboose of a train. The outcome "H" or "T," as well as the time when it occurred in the eyes of the train, meaning the time on the caboose-clock, are written on a slip of paper. This slip is thrown towards the engine at a speed greater than c in the eyes of the ground. This leads to a violation of causality in the eyes of the train, and specifically the engine.

Section III describes quantum entanglement with an analogy between a pair of entangled particles and a pair of magically linked coins. Section IV shows why quantum entanglement does not violate causality.

The basic idea is that either entangled particle can be observed first or, equivalently, may interact with another particle first. In the analogy, either coin can be tossed first. This is the escape hatch. It distinguishes quantum entanglement from the toss of single coin. For a single coin, the toss is the unique cause. It must precede the effect of "Heads" or "Tails." In contrast, with entanglement the cause is not unique. This eliminates the problem with causality that arises for a single coin, namely that the time-order of two events is different in the eyes of the ground and in the eyes of the train.

Entanglement transmits information, rather than a massive, or "physical" body. This lets us break the speed limit of c on a massive body in Special Relativity.

II. A Faster-than-Light Message Violates Causality

First the notation and the idea of two frames of reference are introduced.

For a reader completely unfamiliar with the Theory of Special Relativity, there is little hope, but it might at least be said that Special Relativity flows from the Principle of Relativity: all reference frames are created equal. They all see the same laws of physics. In particular, a blob of light seen by one frame of reference has the same speed c relative to that frame, or in its eyes, as it has in the eyes of any other frame of reference.

This theory teaches that time and distance are not absolute quantities. The time elapsed between two events and the distance between them depend on which frame observes the events. In this theory, a time is the number on the face of a clock and a distance is the number of tick-marks between two positions on a meter-stick. By definition, time and distance are just numbers that come from basic operations of observation and measurement.

Let the ground be one reference frame and a train be the other. In the eyes of the ground, let the train move to the right along the x-axis at constant speed v, with $0 < v < c$ (v lies between 0 and c). The position and time of an event in the ground frame are denoted, respectively, by x and t and in the train frame by x' and t'. The caboose is at the origin, meaning $x' = 0$, in the train frame.

Each frame has fixed observers at rest relative to it. These observers are lined up left and right along the tracks on the ground and from caboose to engine on the train. The observers in each frame have identical clocks at rest next to them. In the eyes of each frame, its own clocks are synchronized.

A set of meter-sticks is at rest in each frame, laid end-to-end along the x-axis. Each frame's observers record the times shown on the clocks and distances shown on the meter-sticks adjacent to them in the other frame as well as their own. They can also compare notes or share data.

Meter-sticks let the observers measure distances. For time, think of stop-watches instead of normal clocks. We only care about how much time elapses between events and a stop-watch can show a time equal to 0, which is useful. It can even show a negative time, which is just some time before the hand or the digital display of the stop-watch reaches 0.

Following tradition, let $\beta \equiv v/c$ and $\gamma \equiv 1/\sqrt{1-\beta^2}$. If the origins of the frames coincide when the clocks at each origin show the time 0, then the Lorentz transform equations are

$$x' = \gamma(x - \beta ct)\,, \tag{1}$$

$$ct' = \gamma(ct - \beta x)\,. \tag{2}$$

These follow logically from the Principle of Relativity. That is not obvious. The Lorentz transform was known before Einstein published on Special Relativity in 1905, but it took him ten years to develop the theory, starting at age sixteen.

The Principle of Relativity is simple and profound. By adopting it, we avoid the unanswerable question, "How does light know how fast to go in empty space?" We assume instead that any given blob of light has the same measured speed in the eyes of any rest-frame, or frame of reference, regardless of their motion relative to each other. This is completely at odds with intuition. It means that distance and time are not what you thought they were before you met Special Relativity.

Equations (1) and (2) take us from the ground's eyes (no primes) to the train's eyes (primes). The space-time coordinates (x, t) are an "event." They can be pictured as standard Cartesian coordinates on perpendicular axes (the "abscissa" and "ordinate") in a two-dimensional space-time plane.

The equations say, "The space-time point or event (x, t) in the ground frame is at the space-time point or event (x', t') in the train frame." The primed points, or events, overlie the unprimed ones in space-time.

The results we need come from Eq. (2). It says that if L is the distance between the caboose and the engine in the ground's eyes, then the engine-clock lags the caboose-clock by the amount $\gamma\beta L/c$ regardless of t.

The second result is that if $x = ct/\beta = (c/\beta)t$ in Eq. (2), then $t' = 0$. This means that some train clock reads $t' = 0$ at any ground time t. It lies farther and farther to the right along the x-axis as time goes by. The x-coordinate of this clock has speed $v_0 \equiv c/\beta > c$. It is a different train clock for every value of t. Although v_0 exceeds c, it need not exceed c by much. If $\beta \to 1$, then $v_0 \to c$. Hence v_0 may exceed c only infinitesimally.

Suppose that the caboose tosses a coin when its own clock reads $t' > 0$ (greater than 0) and writes on a slip of paper both the time t' and the outcome "H" or "T." If the toss is "Heads," then the slip of paper might read, "At caboose time t', the coin landed showing H."

Suppose that this slip of paper is given speed v_m (for "message") along the positive x-axis in the eyes of the ground. If $v_m > v_0$, then this slip of paper will eventually overtake some train clock that reads $t' = 0$. It is moving faster than the coordinate of the train clock that reads $t' = 0$. Note that v_m need only exceed c infinitesimally.

To visualize this, imagine that the train clock that reads $t' = 0$ when the slip of paper arrives is in the engine car at the front of the tran. We have just deduced that when the engine-clock shows the time $t' = 0$, the engine car gets a slip of paper from the caboose about an event that occurred at a time $t' > 0$ in the caboose's eyes. To the train, and in particular the engine, this is a message from the future. It predicts the outcome of a random event, but that event has not yet occurred as far as the engine is concerned.

This means that we have violated causality. Causality says that one cannot know the outcome of a random event yet to occur. One cannot get a slip of paper from the future. Therefore the premise of our argument must have been false. That is, a slip of paper carrying a message cannot exceed the speed limit c. When we assumed that it could, we were led by perfectly logical steps to an absurdity. This is "proof by contradiction" (in Latin, *reductio ad absurdum*).

Of course, nothing like a slip of paper with a positive rest-mass can reach the speed c because, in Special Relativity, a body's mass approaches infinity as its speed approaches c. This theorem is an aside to the main development. Note, however, that even if a slip of paper were massless the argument above says it cannot have a speed greater than c.

III. Quantum Entanglement

When an excited atom "decays" to a state of lower energy, one of its electrons falls to a vacant state of smaller energy, causing the emission of energy in the form of light. The reverse process sees an atom excited by absorbing light, which raises an electron (typically the outermost one) to a state of larger energy.

An atom can decay without changing its rate of spinning, or angular momentum. In some decays, a pair of photons (chunks of light) can be emitted that fly off in opposite directions. Because angular momentum is conserved, the total angular momentum of the two photons is equal to zero. If they were wheels, they would necessarily rotate in opposite directions.

Pairs of electrons can also be emitted from a single process such that their angular momenta add to zero. I will talk about electrons, just for specificity, for the most part.

One should appreciate some basic facts about the angular momentum, or "spin," for brevity, of elementary particles.

To picture angular momentum or spin, align the thumb of your right hand along a vertical axis called z, with z increasing with height. The fingers of your right hand naturally curl in the counterclockwise direction. This goes with counterclockwise rotation and positive angular momentum or "spin" along your thumb and the z-axis. If your thumb points down, the rotation is clockwise. The angular momentum or spin along the z-axis is then negative.

All photons have the same angular momentum or spin, as do all electrons. The spin of a photon is 1 and that of an electron is 1/2 in units of Planck's constant divided by 2π.

The spin of elementary particles is very strange. In particular, the spin of a photon or an electron is always either up or down along any axis that you choose. It does not matter in what direction the axis points. Space is isotropic and spin is quantized. Particles are incomprehensible.

Quantum mechanics says that the first particle to be measured or to interact with another particle does not make up its mind which spin to have until it is measured or interacts with another particle. It is random, indefinite, and unpredictable, and equally likely to be spin-up or spin-down.

As for mathematics, quantum mechanics describes or represents this uncertain two-particle system, prior to being observed or interacting, as the (normalized) sum of the two possible vectors, called "state-vectors," that can result from a measurement or interaction. One method for this uses algebra as follows:

Let $|+>$ represent the up-state and $|->$ the down-state for a single particle. The symbols here are column vectors called “Dirac ket-vectors.” For two entangled particles, say 1 and 2, the possible two-particle states after an observation or interaction may be written $|+,->$ and $|-,+>$. The first and the second symbol are for particle 1 and 2, respectively.

The initial state of the two-particle system, before observation or interaction, is an equal 50-50 mixture of the two states $|+,->$ and $|-,+>$, that is, their sum or difference, with an overall multiplicative constant. Once either particle decides to be spin-up (or spin-down), either because we carried out a measurement on it or because it interacted with a third particle, the other is forced to be spin-down (or spin-up). The state changes and becomes one of the two-particle states. Only one state survives.

That briefly sketches the birth, behavior, and one mathematical representation of entangled particles, but with little discussion or explanation, because the behavior of entangled particles is all we care about. We now can make an analogy between entangled particles and everyday objects.

A pair of entangled particles is analogous to two coins that are minted simultaneously by a method that magically links them. They fly apart, but by magic if one lands showing “Heads,” then the other will land showing “Tails.” It does not matter how far apart the coins are, either can be tossed first, the outcome of the first toss is random, and the second coin can be tossed immediately after the first.

This is nearly unbelievable. Does nature allow this? Nothing could be weirder. A message or signal goes from the first coin to the second instantaneously, telling it what to do. This holds no matter how far apart they are, no matter what lies betweeen them, and with no physical connection between them. A similar problem, if not the same one in different garb, occurs with the "collapse of the wave function."

The quantum mechanical wave function of any one electron contains everything that can be said about it. It varies with position and time, and, left on its own, extends throughout all space. It does not vanish (equal zero) anywhere. It specifies the likelihood, or more precisely the probability per cubic centimeter, that the electron will appear at any given location at any given time. Einstein did not believe nature was at heart random, or probabilistic.

If an electron is found in a tiny volume, then its wave function collapses, becoming highly localized. That is, outside of that volume, it is zero. How does the part of the wave function near Uranus or in the Horsehead Nebula know that it has to vanish in time for the wave function to collapse? Texts ignore this obvious question.[5–8]

Has the information that a measurement or interaction is about to occur flown over an infinite distance in no time at all? Is everything foreordained? What is going on? No one knows.

IV. Causality is Preserved

To be clear from the start, the question is whether any observer sees the effect of a cause prior to the cause. Is the proper time-order of cause and effect upset in the eyes of some reference frame? The question is not whether quantum entanglement is a real phenomenon nor whether a message can travel infinitely fast. It can.

Let one entangled coin be in the caboose and the other in the engine. Suppose the caboose tosses its coin first, in the eyes of the ground, when its own clock reads $t' = 0$. If it lands showing "Heads," then the engine-coin must land showing "Tails."

As seen by the ground frame, the engine-clock lags the caboose-clock. When the caboose-clock reads $t' = 0$, the engine-clock reads $-\gamma\beta L/c$, where L is the length of the train in the ground's eyes.

It might seem from this that the engineer receives a faster-than-light message from the caboose, and from the future, before the engine-coin is tossed, that predicts the outcome of that toss. This seems to violate causality. It seems as if the engineer got a slip of paper, in effect, as in Section II.

It is not that way at all. The engineer does not get a slip of paper. Instead, the coin receives the message. The engineer does not know that the outcome of the coin-toss in the engine is determined.

Recall again that, in the eyes of the ground, the engine-coin may be tossed immediately after the coin-toss in the caboose. Suppose it were. In the eyes of the train, *the engine-toss would precede the caboose-toss*, because the engine-clock lags the caboose-clock in the eyes of the ground. The engine-clock shows a negative time.

This means that, in the eyes of the train, the coin-toss in the engine is the cause of the outcome of the coin-toss in the caboose. To the train, this outcome is the effect, and the cause would precede the effect. Causality is not violated. There is no paradox.

Although the engine-coin may be tossed before the engine-clock reads $t' = 0$, that is, for some value $t' < 0$, it need not be tossed. The toss could be delayed indefinitely. Regardless of when or whether it is tossed, the outcome of its toss is decided before the engine-clock reads $t' = 0$. It was decided when the engine-clock read $-\gamma\beta L/c$.

Suppose instead that, in the ground's eyes, the engine-coin is tossed before the caboose-coin. Then the outcome of the coin-toss in the caboose is dictated. To the ground, the engine-toss is the cause and the outcome of the caboose-toss is the effect. Here the time-ordering is the same in the train's eyes, because the engine-clock lags the caboose-clock. That is, in the train's eyes, the engine-toss precedes the caboose-toss. As a specific case, suppose that, in the ground's eyes, the engine-coin is tossed when the engine-clock reads $t' = 0$. Then the caboose-clock reads $\gamma\beta L/c$. Before that time, the outcome of the caboose-toss is random.

Causality cannot be violated since neither toss is the unique cause of the outcome of the other toss. To the ground, the coin-toss in the caboose may be the cause, dictating the result of that in the engine, the effect. To the train, in the very same experiment, the coin-toss in the engine may be the cause, dictating the result of that in the caboose, the effect. In both frames, cause precedes effect, despite the infinite speed of the signal.

V. Remarks

The proof used the fact that the engine-clock lags the caboose-clock in the eyes of the ground. This follows directly from the Principle of Relativity. As usual, we consider a specific thought-experiment.

Imagine that the caboose and engine synchronize their clocks by placing a flashbulb midway between them, in the dining car, say. The bulb simultaneously emits a light pulse going left towards the caboose and another going right towards the engine. The caboose-clock and the engine-clock start to tick, reading time $t' = 0$, when their respective pulses reach them. To the train, the two pulses have speed c and arrive simultaneously at the caboose and engine.

The ground frame disagrees. To those observers, the right-going pulse travels farther to reach the engine than the left-going pulse travels to reach the caboose because the train is going to the right.

In the ground's eyes, each pulse has speed c, and therefore the left-going pulse reaches the caboose before the right-going pulse reaches the engine. Hence the caboose-clock begins ticking before the engine-clock, which is the same as saying that the caboose-clock leads or shows a later time than the engine-clock. Equivalently, the engine-clock lags the caboose-clock. As mentioned, the amount of lag is constant.

In Reference 9, Feynman discusses the EPR paper. He is not distressed. He also discusses charge conservation to illustrate a local conservation law. Reference 10 has Bell's articles. Reference 11 is a popular book, meaning for the general public. Marburger describes quantum entanglement as "deeply disturbing."[12] He does not say, "deeply distressing" nor dwell on it.

An aside: we saw that simultaneity is lost in Special Relativity. That is, in Eq. (2), for a fixed value of t, t' is a different number at different values of x. In the ground's eyes, the train's clocks are not synchronized. One cannot freely play games with time and space, however. In particular, if two events A and B are "causally related" in the ground frame, then they are also causally related in the train frame.

"Causally related" means this: if A and B are separated by a distance x and by a time t in the ground frame, then $x \leq ct$. This is the same as $x/t \leq c$, which says that one event can send a message to the other in an elapsed time of t at a speed less than or equal to c. Equivalently, if a cause propagated at speed less than or equal to c, then one event could cause an effect at the other in time less than or equal to t.

For "acausal events," $x > ct$. Such events are too far apart for a message to go from one to the other in time t, even at speed c. One event cannot cause the other, unless a cause propagates at a speed greater than c. This happens with entangled particles.

Using Eqs. (1) and (2) to prove the claim above regarding causal events in different frames is a very good (not too hard) high-school algebra problem. Similar algebra problems are to show that a) a clock ticks more slowly (emits fewer ticks per second) when moving than when stationary and b) a train is shorter in length when moving than when stationary, in both cases by a factor of γ.

Section IV noted that, in the special case of quantum entanglement, one cannot say which of two events is the cause of the other in an absolute sense. Instead, it depends on which rest-frame observes them. Nature has therefore lost the uniqueness of the arrow of time. This may suggest questions about entropy, among other topics.

Entanglement information seems to have zero rest-mass. If it did not, then its mass would become infinite (or imaginary) when it reached (or exceeded) c, according to the Theory of Special Relativity.

You might object to the proof, saying, "Entangled particles cannot ride in the caboose and engine. They always move with opposite velocities, away from their birthplace, not at the same velocity as a train."

The particles or coins were riding in the caboose and engine to make the experiment easier to picture. Their speeds do not matter. They could have been moving with opposite velocities in the ground's eyes, for example, not at rest relative to the train. I assumed only that the particles or coins revealed their states when they happened to be adjacent to the caboose and engine.

Again, as mentioned, the first particle to be observed or to interact with another particle decides randomly and on its own which message to send. The experimentalist can force the particle to decide, but is not a party to its choice. This means that one cannot send a “1” or “0” at will via the magic link.

The link is delicate. You cannot even breath on it. You should avoid looking at it or even thinking too much about it. If you touch it, it is destroyed.

Such behavior, however, is typical of all coherent quantum mechanical effects. For example, if a stream of monoenergetic electrons is directed at an opaque plate with two slits in it, then on a screen behind the plate the number of electrons arriving varies with position in a coherent wave-like interference or diffraction pattern, exactly like the bright and dark fringes in optical diffraction. It is as if the electrons were sound waves or light waves, not particles. The wavelength is inversely proportional to the electrons’ momentum, according to de Broglie’s formula.

This happens even if the electrons are fired one at a time at the plate. It would take longer to get the same statistics, but this means that the interference pattern is a not collective effect, something produced by electrons interacting with each other. Instead, each electron acts like a wave.

The pattern disappears if the experiment is designed to determine which slit each electron went through. That determination requires an interaction with each electron, which upsets its wavefunction. It is no longer a plane wave.

An example of the difference between coherent waves with very flat wavefronts and other waves is the twinkle of stars and steady glow of planets. Stars are so distant that their wavefronts are flat to within less than one wavelength of light. Planets are close enough that their light comes from slightly different directions, so that it is not a plane wave. The result is that random variations in atmospheric conditions have no significant effect on the brightness of a planet, but produce strong interference when they scatter starlight.

Another example is the ever-shifting, bright and dark speckle pattern of a spot of light from a laser. The interference pattern results from the nearly perfect periodicity of the light beam.

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3. D.Z. Albert and R. Galchen, "A Quantum Threat," *Sci. Am.*, March, 2009, p. 32-39. The authors claim that the Theory of Special Relativity is threatened by quantum entanglement.

4. R. Resnick, *Introduction to Special Relativity*, John Wiley & Sons, New York (1968). Problem 39, page 106 reads: "Suppose that an event A causes event B in frame S, the effect being propagated with a speed *greater than c*. Show, using the velocity addition theorem, that there exists an inertial frame S', which moves relative to S with a velocity less than c, in which the order of these events would be reversed. Hence, if the concepts of cause and effect are to be preserved, it is impossible to send signals with a speed greater than that of light." Today, in light of entanglement, we know that it is impossible for us to send signals with a speed greater than c without violating causality, but entangled particles can.

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10. J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press, New York (1993). According to Reference 11, this contains Bell's papers.

11. Amir D. Aczul, *Entanglement: The Greatest Mystery in Physics*, Penguin Group, Plume, New York (2003).

12. J.H. Marburger, III, *Constructing Reality*, Cambridge University Press, Cambridge (2011), pages 144-152. This is an excellent popular treatment of the Standard Model, the quantum theory of fields and particles. Marburger gives a variation of Bell's inequality different from that in Reference 1 with a shorter argument. He also notes that the Standard Model is not a model; no one can visualize how quantum mechanics actually works. He notes that

the word "particle" for electrons and other particles is a misnomer. In a sense, any particle is all particles. The book would be worth the cost for the anecdotes in the footnotes. Marburger's article "What Is a Photon?" [*The Physics Teacher* **34**(8), 482 (1996)] is also excellent.